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DETERMINING THE CKM UNITARITY TRIANGLE FROM B DECAYS TO CHARGED PIONS AND KAONS[1]

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ABSTRACT

Decay rates of $B^0(t) \to \pi^+\pi^-$, $B^0 \to \pi^- K^+$, $B^+ \to \pi^+ K^0$ $(K_S \to \pi^+\pi^-)$ and of charge-conjugate processes are studied within flavor SU(3) symmetry and first-order SU(3) breaking. We show that these measurements can determine with a reasonable accuracy the two angles, α and γ, of the Cabibbo-Kobayashi-Maskawa unitarity triangle.

B decays provide a variety of CP asymmetry measurements [1], which can test the currently favored hypothesis that phases in elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2] are the source of the observed CP violation in the neutral kaon system [3]. The time-dependent rate-asymmetry between the process $B^0(t) \to \pi^+\pi^-$ and its CP-conjugate measures one of these phases, the angle α of the CKM unitarity triangle. Penguin amplitudes [4] and higher order electroweak penguin contributions [5] complicate the situation somewhat. However, by measuring also the rates of $B^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^0$ and of their charge-conjugate counterparts one can isolate the amplitudes contributing to final states with isospin 0 and 2 and thereby determine α with a rather good accuracy [6, 7]. The detection of the modes involving neutral pions poses an interesting challenge for future experiments.

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A few alternative ways to learn the penguin effect in $B^0 \to \pi^+\pi^-$ were suggested recently. DeJongh and Sphicas [8] have studied in detail the dependence of the asymmetry in $B^0(t) \to \pi^+\pi^-$ on the (unknown) magnitude and relative phase of the tree and penguin amplitudes contributing to this process. Using flavor SU(3) symmetry, Silva and Wolfenstein [9] proposed to approximately estimate the penguin contribution by comparing the tree-dominated decay rate of $B^0 \to \pi^+\pi^-$ with that of $B^0 \to \pi^- K^+$ which has a large penguin term. Buras and Fleischer [10] suggested to isolate the penguin term in $B^0 \to \pi^+\pi^-$ from its (SU(3)-related) dominant effect in the time-dependent asymmetry of $B^0(t) \to K^0\overline{K}^0$.

In the present report we describe a method which determines simultaneously both the angle α and the angle γ of the unitarity triangle from the decay rates of $B^0(t) \to \pi^+\pi^-$, $B^0 \to \pi^- K^+$, $B^+ \to \pi^+ K^0$ (where $K^0 \to K_S \to \pi^+\pi^-$) and their charge-conjugates. All these modes are detected by charged pions and kaons in the final state. Other ways to measure γ, based on charged B decays, were proposed in Ref. [11]. Our method employs flavor SU(3) symmetry [12, 13, 14], and neglects "annihilation" amplitudes in which the spectator quark (the light quark accompanying the b in the initial meson) enters into the decay Hamiltonian [15]. These amplitudes in B decays are expected to be suppressed by f_B/m_B, where $f_B \simeq 180$ MeV. In order to improve the precision of the method, we also include first-order SU(3) breaking terms [16]. Second order corrections, which are expected to be at a level of a few percent, will be neglected.

In the SU(3) limit and neglecting annihilation terms all B decay amplitudes into $\pi\pi$, πK and $K\overline{K}$ states can be decomposed in terms of three independent amplitudes [7, 15]: a "tree" contribution $t(t')$, a "color-suppressed" contribution $c(c')$ and a "penguin" contribution $p(p')$. These amplitudes contain both the leading-order and electroweak penguin contributions:

$$t \equiv T + (c_u - c_d)P^C_{EW} \quad , \quad c \equiv C + (c_u - c_d)P_{EW} \quad , \quad p \equiv P + c_d P^C_{EW} \quad . \tag{1}$$

Here the capital letters denote the leading-order contributions defined in Ref. [15], and P_{EW} and P^C_{EW} are color-favored and color-suppressed electroweak penguin amplitudes defined in Ref. [7]. The values $c_u = 2/3$ and $c_d = -1/3$ are those which would follow if the electroweak penguin coupled to quarks in a manner proportional to their charges. (Small corrections, which we shall ignore and which do not affect our analysis, arise from axial-vector Z couplings and from WW box diagrams.) The $\Delta S = 0$ amplitudes are denoted by unprimed quantities and the $\Delta S = 1$ processes by primed quantities.

The amplitudes of the two processes $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^- K^+$ are expressed as

$$\begin{aligned} A_{\pi\pi} &\equiv A(B^0 \to \pi^+\pi^-) = -t - p = -T - P - \frac{2}{3}P^C_{EW} \quad , \\ A_{\pi K} &\equiv A(B^0 \to \pi^- K^+) = -t' - p' = -T' - P' - \frac{2}{3}P'^C_{EW} \quad , \end{aligned} \tag{2}$$

while that for $B^+ \to \pi^+ K^0$ will be approximated by

$$A_+ \equiv A(B^+ \to \pi^+ K^0) = p' = P' - \frac{1}{3}P'^C_{EW} \approx P' + \frac{2}{3}P'^C_{EW} \quad , \tag{3}$$

neglecting a color-suppressed electroweak penguin effect of order $|P'^C_{EW}/P'| = \mathcal{O}((1/5)^2)$ [7]. With this approximation, A_+ contains the same combination of electroweak and gluonic penguins as in the expression for $A_{\pi K}$.

The terms on the right-hand-sides of (2) and (3) carry well-defined weak phases. The weak phase of T is $\mathrm{Arg}(V_{ud}V^*_{ub}) = \gamma$, and that of $P + \frac{2}{3}P^C_{EW}$ is approximately $\mathrm{Arg}(V_{td}V^*_{tb}) = -\beta$, where we neglect corrections due to quarks other than the top quark. The effects of the u and c quarks become appreciable [17] when V_{td} obtains its currently allowed smallest values. This corresponds to a small deviation of the CP asymmetry in $B^0(t) \to \pi^+\pi^-$ from $\sin(2\alpha)\sin(\Delta mt)$ (where Δm is the neutral B mass-difference). For large values of V_{td}, where the deviation due to the penguin amplitude becomes significant [18], the u and c contributions become very small. T' also carries the phase γ, while the weak phase of $P' + \frac{2}{3}P'^C_{EW}$ is $\mathrm{Arg}(V_{ts}V^*_{tb}) = \pi$. The ratio of $\Delta S = 1$ to $\Delta S = 0$ tree and penguin amplitudes are given by the corresponding ratios of CKM factors, $|T'/T| = |V_{us}/V_{ud}| \equiv r_u = 0.23,\ |P'/P| = |V_{ts}/V_{td}| \equiv r_t$.

Denoting $\mathcal{T} \equiv |T|,\ \mathcal{P} \equiv |P + \frac{2}{3}P^C_{EW}|$ and assigning SU(3)-symmetric strong phases $\delta_T,\ \delta_P$ to terms with specific weak phases, (2) and (3) may be transcribed as

$$\begin{aligned} A_{\pi\pi} &= \mathcal{T}e^{i\delta_T}e^{i\gamma} + \mathcal{P}e^{i\delta_P}e^{-i\beta} \quad , \\ A_{\pi K} &= r_u\mathcal{T}e^{i\delta_T}e^{i\gamma} - r_t\mathcal{P}e^{i\delta_P} \quad , \\ A_+ &= r_t\mathcal{P}e^{i\delta_P} \quad . \end{aligned} \tag{4}$$

To introduce first-order SU(3) breaking corrections, we note that in the T' amplitude the W turns into an $\overline{s}$ quark instead of a $\overline{d}$ in T. This SU(3) breaking term was denoted by T'_1 in Ref. [16]. Assuming factorization for T, which is supported by experiments [19], SU(3) breaking is given by the K/π ratio of decay constants

$$\frac{\mathcal{T}'}{\mathcal{T}} = \frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} \equiv \tilde{r}_u \quad . \tag{5}$$

In the penguin amplitudes (including electroweak penguin) of both $B^0 \to \pi^-K^+$ and $B^+ \to \pi^+K^0$ the $\overline{b}$ quark turns into an $\overline{s}$ quark instead of a $\overline{d}$ in $B^0 \to \pi^+\pi^-$. This SU(3) breaking term was denoted by P'_1 in Ref. [16]. Here we will denote the magnitude of the $\Delta S = 1$ penguin amplitude by $r_t\tilde{\mathcal{P}}$, to allow for SU(3) breaking. Since factorization is questionable for penguin amplitudes, one generally expects $\tilde{\mathcal{P}} \neq (f_K/f_\pi)\mathcal{P}$. We will assume that the phase δ_P is unaffected by SU(3) breaking. Since this phase is likely to be small [20], this assumption is not expected to introduce a significant uncertaintly in the determination of the weak phases.

Thus, including first-order SU(3) breaking, Eqs. (4) are modified to become

$$\begin{aligned} A_{\pi\pi} &= \mathcal{T}e^{i\delta_T}e^{i\gamma} + \mathcal{P}e^{i\delta_P}e^{-i\beta} \quad , \\ A_{\pi K} &= \tilde{r}_u\mathcal{T}e^{i\delta_T}e^{i\gamma} - r_t\tilde{\mathcal{P}}e^{i\delta_P} \quad , \\ A_+ &= r_t\tilde{\mathcal{P}}e^{i\delta_P} \quad . \end{aligned} \tag{6}$$

It will be shown that the numerous *a priori* unknown parameters in (6), including the two weak phases $\alpha \equiv \pi - \beta - \gamma$ and γ, can be determined from the rate measurements of the above three processes and their charge-conjugates.

First, we note that the amplitudes for the corresponding charge-conjugate decay processes are simply obtained by changing the signs of the weak phases γ and β. We denote the charge-conjugate amplitudes corresponding to (6) by $\overline{A}_{\pi\pi},\ \overline{A}_{\pi K},\ A_-$, respectively.

The time-dependent tagged B^0 and $\overline{B}^0$ decay rates to $\pi^+\pi^-$ are given by

$$\begin{aligned}\Gamma(B^0(t) \to \pi^+\pi^-) &= e^{-\Gamma t}[|A_{\pi\pi}|^2 \cos^2(\frac{\Delta m}{2}t) + |\overline{A}_{\pi\pi}|^2 \sin^2(\frac{\Delta m}{2}t) \\ &\quad + \mathrm{Im}(e^{2i\beta} A_{\pi\pi} \overline{A}^*_{\pi\pi}) \sin(\Delta m t)] \quad , \\ \Gamma(\overline{B}^0(t) \to \pi^+\pi^-) &= e^{-\Gamma t}[|A_{\pi\pi}|^2 \sin^2(\frac{\Delta m}{2}t) + |\overline{A}_{\pi\pi}|^2 \cos^2(\frac{\Delta m}{2}t) \\ &\quad - \mathrm{Im}(e^{2i\beta} A_{\pi\pi} \overline{A}^*_{\pi\pi}) \sin(\Delta m t)] \quad . \end{aligned} \tag{7}$$

Measurement of these rates determines $|A_{\pi\pi}|^2,\ |\overline{A}_{\pi\pi}|^2$ and $\mathrm{Im}(e^{2i\beta} A_{\pi\pi} \overline{A}^*_{\pi\pi})$:

$$\begin{aligned}|A_{\pi\pi}|^2 &= \mathcal{T}^2 + \mathcal{P}^2 - 2\mathcal{T}\mathcal{P}\cos(\delta - \alpha) \quad , \\ |\overline{A}_{\pi\pi}|^2 &= \mathcal{T}^2 + \mathcal{P}^2 - 2\mathcal{T}\mathcal{P}\cos(\delta + \alpha) \quad , \\ \mathrm{Im}(e^{2i\beta} A_{\pi\pi} \overline{A}^*_{\pi\pi}) &= -\mathcal{T}^2 \sin(2\alpha) + 2\mathcal{T}\mathcal{P}\cos\delta \sin\alpha \quad , \end{aligned} \tag{8}$$

where we used $\beta + \gamma = \pi - \alpha$ and where we defined $\delta \equiv \delta_T - \delta_P$. The rates of the self-tagging modes $\pi^- K^+$, $\pi^+ K^-$ and $\pi^+ K^0$ determine $|A_{\pi K}|^2$, $|\overline{A}_{\pi K}|^2$ and $|A_+|^2$, respectively:

$$\begin{aligned}|A_{\pi K}|^2 &= \tilde{r}_u^2 \mathcal{T}^2 + r_t^2 \tilde{\mathcal{P}}^2 - 2\tilde{r}_u r_t \mathcal{T}\tilde{\mathcal{P}}\cos(\delta + \gamma) \quad , \\ |\overline{A}_{\pi K}|^2 &= \tilde{r}_u^2 \mathcal{T}^2 + r_t^2 \tilde{\mathcal{P}}^2 - 2\tilde{r}_u r_t \mathcal{T}\tilde{\mathcal{P}}\cos(\delta - \gamma) \quad , \\ |A_+|^2 = |A_-|^2 &= r_t^2 \tilde{\mathcal{P}}^2 \quad . \end{aligned} \tag{9}$$

Measurement of the six quantitities in (8)$-$(9) suffices to determine all six parameters $\alpha,\ \gamma,\ \mathcal{T},\ \mathcal{P},\ \tilde{\mathcal{P}},\ \delta$ up to discrete ambiguities. The CKM parameter $r_t \equiv |V_{ts}/V_{td}|$, which is still largely unknown, is obtained from the unitarity triangle in terms of α and γ:

$$r_u r_t = \frac{\sin\alpha}{\sin\gamma} \quad . \tag{10}$$

We note immediately that

$$|A_{\pi K}|^2 - |\overline{A}_{\pi K}|^2 = -(\frac{f_K}{f_\pi})(\frac{\tilde{\mathcal{P}}}{\mathcal{P}})(|A_{\pi\pi}|^2 - |\overline{A}_{\pi\pi}|^2) \quad , \tag{11}$$

which determines the magnitude of SU(3) breaking in the penguin amplitude, $\tilde{\mathcal{P}}/\mathcal{P}$. The relation (11) between the particle-antiparticle rate differences in $B \to \pi K$ and in $B \to \pi\pi$ was recently derived [21] in the SU(3) limit, $f_K/f_\pi \to 1$, $\tilde{\mathcal{P}}/\mathcal{P} \to 1$. The authors assumed for SU(3) breaking a value $\tilde{\mathcal{P}}/\mathcal{P} = f_K/f_\pi$ (based on factorization of

penguin amplitudes) which is questionable. In our approach this ratio is a free parameter to be determined by experiment. We expect it to differ from one by up to 30%.

A combined sample of the decays $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^- K^+$ has already been observed [22] with a joint branching ratio of about 2×10^{-5}. Equal mixtures of the two modes are likely, although confirmation of this estimate awaits a better π/K separation. A similar branching ratio is expected for $B^+ \to \pi^+ K^0$, where the efficiency of observing a K^0 by a K_S decay to two charged pions is 1/3. Samples of hundreds of events in each of these modes (combining $B^+ \to \pi^+ K^0$ and $B^- \to \pi^- \overline{K}^0$) are expected to be obtained in future e^+e^- colliders operating at the Υ(4S) resonance. The resulting statistical accuracy of determining the weak phases α and γ using the above method thus is expected to be at a level of ten percent. The theoretical uncertainty of the method is at a similar level, involving the following corrections all of which are of order a few percent: A correction from an electroweak penguin amplitude in $B^+ \to \pi^+ K^0$, corrections due to u and c quarks in the $B^0 \to \pi^+\pi^-$ penguin amplitude, second-order SU(3) breaking in the magnitudes of weak amplitudes, and first order SU(3) breaking in the (small) strong phase of the penguin amplitude.

To summarize, we have shown that measurements of the rates for B decays to modes involving charged pions and kaons in the final states can determine the shape of the unitarity triangle. The accuracy of this method of determining the angles α and γ in future e^+e^- B factories is roughly estimated to be at a level of 10%. More detailed studies of the precision of this method are worthwhile.

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